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Divergence Boundary Prediction from Random Responses: NAL's Method

Yuji Matsuzaki* and Yasukatsu Ando†
National Aerospace Laboratory, Tokyo Japan

Introduction

IN Refs 1 and 2 we presented the National Aerospace Laboratory (NAL) method which can predict both flutter and divergence boundaries in an unified manner from turbulence excited responses at subcritical speeds The estimation method[3] consists of the following procedures A time series of the response of the aeroelastic system to the Gaussian random noise input is represented by an autoregressive moving average (Æ MA) process Instead of the Æ MA process an autoregressive (Æ) process may also be used The order and coefficients of the process are estimated by Akaike's AIC minimum procedure [4] Stability of the system is evaluated with the aid of Jury's stability criteria[5] in which the estimated Æ coefficients are used The flutter or divergence boundary is determined by a least squares fit of a straight line or a parabolic curve to each set of stability parameters plotted against the dynamic pressure In addition to the boundary the modal frequencies and damping ratios are calculated from the Æ coefficients

As an example of possible application the method was applied to response signals of a cantilever wing model measured in a low supersonic, subcritical flutter test The comparison between the acutal and estimated flutter boundaries showed that an accurate prediction could be made using the data obtained in a narrow dynamic pressure range sufficiently below the boundary It was also demonstrated that no prediction could be made with the aid of the conventional method based on the damping ratio if one resorted to only a set of the estimated damping ratios obtained in the same narrow range of the dynamic pressure [1 2 6]

The objective of this brief Note is to show that the NAL estimation method is also applicable to subcritical divergence testing In most cases divergence is the aeroelastic mode of instability for a forward swept wing Recent progress in advanced composite materials makes it now possible for aeronautical engineers to design aircrafts having wings with no serious weight penalty However little experimental information on divergence has been accumulated to date Therefore it is important to develop an accurate rapid and low cost technique for predicting the divergence boundary and characteristics without destruction of expensive tailored composite models during testing

Ricketts and Doggett[7] performed divergence tests on flat aluminum plate wings at transonic speeds to apply four static and two dynamic techniques and evaluate their accuracy in prediction the divergence dynamic pressure The static methods required a process of stepping the model through an angle of attack range and acquiring mean strains at each angle while keeping the flow constant It was necessary to repeat this process for several dynamic pressures in order to predict the divergence boundary In the dynamic methods a spectrum analyzer was used to determine the modal frequency and peak amplitude from the turbulence excited responses of the models fixed at zero angle of attack According to Ricketts and Dogget's evaluation the former are more reliable and accurate than the latter This is because all the static techniques are more elaborate They are however more time consuming

The NAL method is classified as a dynamic approach in which the estimation of aeroelastic characteristics of wings is based on an advanced time series analysis theory

Model and Test Procedure

Although three wing models were tested we will focus here on one of them because the results for all three models are similar The model was constructed of an aluminum alloy flat plate of 4 mm thickness The plate was double wedged at the leading and trailing edges Figure 1 shows the wing planform strain gauge position first three natural frequencies and nodal lines of the second and third modes The experiment was conducted in the blowdown type supersonic wind tunnel of the 1×1 m test section at the NAL The strain was measured at fourteen dynamic pressures from $Q = 0\ 716$ to $1\ 015\ kg/cm^2$ with the Mach number fixed at 2 5 The signal was recorded on an FM magnetic recorder After completion of the 14 measurements at constant flow conditions the dynamic pressure was increased until divergence actually occurred in order to determine the divergence dynamic pressure $Q_D = 1\ 085\ kg/cm^2$ The dynamic pressure was increased at a rate of 0 026 $kg/cm^2/s$

Data Analysis and Results

Data analysis of the subcritical response signals for divergence is essentially the same as that for the flutter

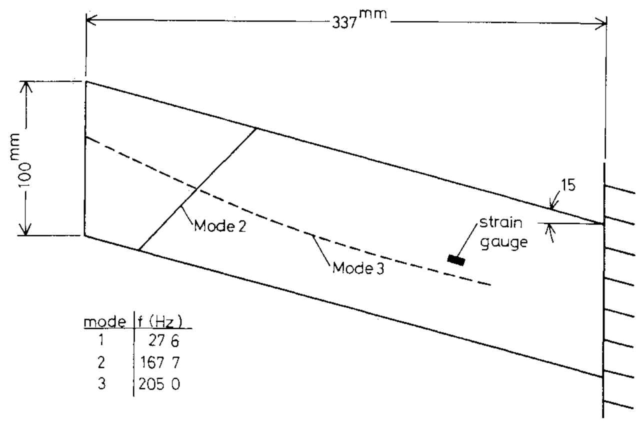

Fig 1 Forward swept wing configuration strain gauge location natural frequencies and nodal lines

Received May 21 1983; revision received Jan 11 1984

*Principal Research Engineer Member AIAA

†Principal Research Engineer

Table 1 Comparison of estimated divergence boundaries ($\hat{Q}_D/Q_D{}^a$)

No of data K	6	7	8	9	10	11	12	13	14
Range of Q/Q_D % (from 66% of Q_D)	~75	~77	~79	~82	~85	~86	~89	~91	~94
$\hat{Q}_D/Q_D$, %	89	93	96	99	99	101	102	103	103

[a] $Q_D = 1\ 085$ kg/cm^2 actual divergence dynamic pressure

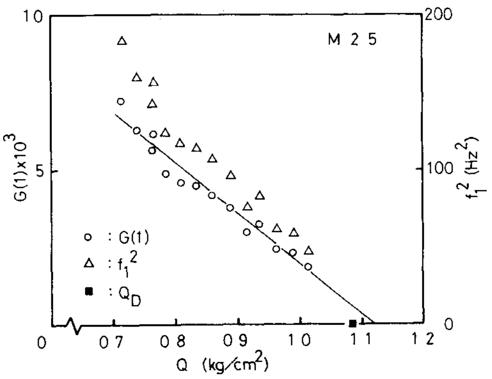

Fig 2 $G(1)$, f_1^2 and estimation of divergence point by extrapolation of a straight line fitted to all circles

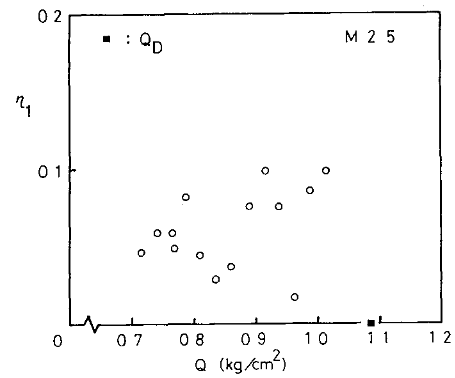

Fig 3 Estimated damping ratio of the first mode

described in Refs 1 2 3 and 6 According to a preliminary spectrum analyzer examination the first mode component contained in the signals was so predominant that the spectrum of the higher modes was not detectable in most cases As for the first mode the frequency decreased from about 14 to 7 Hz as the dynamic pressure increased from 0 716 to 1 015 kg/cm^2 In order to make a one mode analysis for all the cases the signals were narrowed by a band pass filter with a frequency range of 3 to 30 Hz

A discrete time series $\{y(n)\}$ $n=1$ N was generated by sampling the band passed signals at a time interval $T=0\ 001$ s The number of the points and the maximum lag of covariance used in the analysis were $N=4096$ and $k_{\max}=2\sqrt{N}=128$ respectively The Akaike minimum AIC procedure showed that the estimated order of the *AR* part was second for all the signals analyzed that is $J=1$ in Eq (1) or $n=2$ in Eq (14) of Ref 1 Among the stability parameters $G(\pm 1)$ and Ff7 (1) it was only $G(1)$ that had a tendency to decrease with increasing synamic pressure Therefore the instability en countered should be a static instability as shown in Ref 1 The values of $G(1)$ are plotted by circles against the dynamic pressure in Fig 2 For comparison the measured dynamic pressure Q_D is represented by a solid square on the coor dinate The divergence boundary was estimated to be 103% of Q_D by an extrapolation of the least squares fit of a straight (solid) line to all the circles Table 1 shows the change in accuracy of estimation with increasing data points K used The estimated to actual pressure ratio (%) was calculated by using the data points of the lowest K dynamic pressures ($K=6$ to 14) It is noted that the estimation becomes non conservative if the points at the pressures close to Q_D are included in the line fitting

Next the estimated frequency f_l and damping ratio η_l of the first mode are briefly discussed The squared values of the frequency f_l^2 are also plotted in Fig 2 There is a strong correspondence between f_l^2 and $G(1)$ It should be noted in passing that this is not necessarily always the case The result for one of two other wings showed that such a corresponding relation was lost at one dynamic pressure where the estimated damping was greatly deviated from those at the remaining dynamic pressures The estimation based on f_l^2 gives a similar boundary as determined by $G(1)$ As for the damping ratio estimated values are quite scattered between 0 01 and 0 1 shown in Fig 3 No trend with increasing dynamic pressure is observed

Concluding Remarks

The NAL method for estimation of the aeroelastic in stability boundary from randomly excited responses has been demonstrated to be also effective for subcritical divergence testing of a cantilever wing model performed at a supersonic wind tunnel In the one mode analysis it is shown that the stability parameter $G(1)$ of Jury's criteria governing static instability is closely related to the squared value of the frequency of the mode losing the stability

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